Statistics

Summer 2023

Lecture 11

Feb 19-8:47 AM

Class QZ 13

Given a binomial Prob. dist with

$n=250$ & $P=.8$

1) $q=1-P=\boxed{.2}$ ✓

2) $\mu=np=\boxed{200}$ ✓

3) $\sigma^2=npq=\boxed{40}$ ✓

4) σ (Round to whole #)

$=\sqrt{\sigma^2}=\sqrt{40}\approx\boxed{6}$ ✓

5) $\overbrace{\text{Usual Range}}^{\text{95\% Range}}$

$\mu\pm2\sigma=200\pm2(6)$

$\Rightarrow\boxed{188 \text{ to } 212}$ ✓

Reduce by 1

6) $P(195\le x\le 210)$

$=\text{binomcdf}(250,.8,210)-\text{binomcdf}(250,.8,194)=\boxed{.763}$ ✓

Jun 28-11:14 AM

Data
- 1) Qualitative
  (Non-Numerical)
- 2) Quantitative
  (Numerical)
  - 1) Discrete
    Countable
  - 2) Continuous
    Measureable

Jun 28-11:03 AM

We use Continuous random variable (SG 18-21)

1) Uniform Prob. dist.

2) Standard normal Prob. dist.

3) Normal Prob. dist.

4) Central Limit Theorem

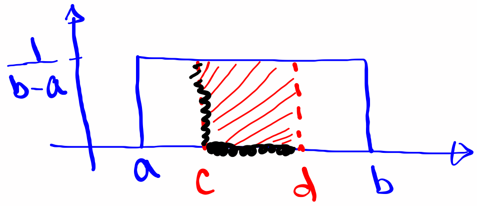
5) Applications

Jun 28-11:05 AM

Uniform Prob. Dist.

Let x be a continuous random variable for all values from a to b with uniform Prob. dist.

1) Graph is rectangular from a to b with width = $\frac{1}{b-a}$



$P(c<x<d)$

$= (d-c)\cdot \frac{1}{b-a}$

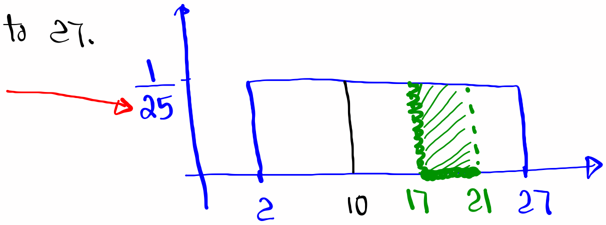
$P(x=c)=0$

Line → Zero Area

Jun 29-7:35 AM

Consider a uniform Prob. dist. for all values from 2 to 27.

$27-2$



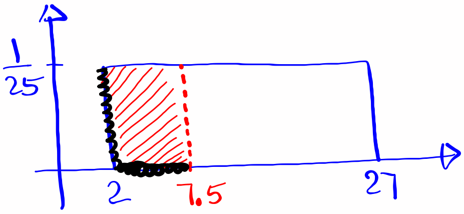
$P(x=10)=0$

$P(17<x<21) = (21-17)\cdot \frac{1}{25} = \frac{4}{25}$

$P(x<7.5)$

$= (7.5-2)\cdot \frac{1}{25}$

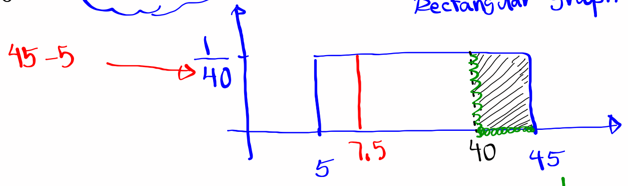
$= \frac{5.5}{25} = \boxed{\frac{11}{50}}$



Jun 29-7:40 AM

Let x be a cont. random variable for all values from $\boxed{5 \text{ to } 45}$ with uniform Prob. dist

Rectangular Graph



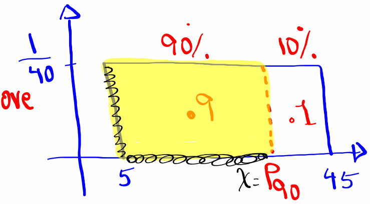
1) $P(x=7.5)=0$

2) $P(x>40)=(45-40)\cdot\frac{1}{40}$

$=\frac{5}{40}=\boxed{\frac{1}{8}}=\boxed{.125}$

3) find $x=P_{90}$

90% below & 10% above



$(x-5)\cdot\frac{1}{40}=.9$

multiply by 40

$x-5=40(.9)$

$x-5=36$

$x=36+5$ $\boxed{x=41}$

Jun 29-7:46 AM

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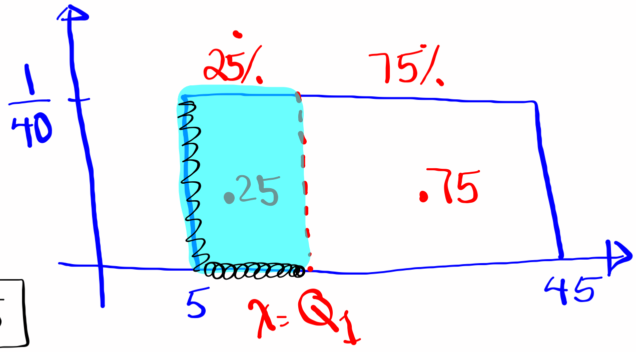
4) find $x=Q_1$

25% below & 75% above



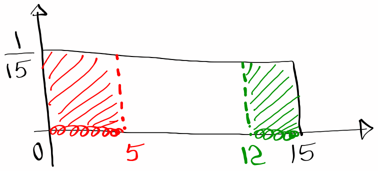
$(x-5)\cdot\frac{1}{40}=.25$

$x-5=40(.25)$

$x-5=10$ $\boxed{x=15}$

Jun 29-7:53 AM

Wait time to be Seated at a local food place is $\overset{0 \le x \le 15}{\boxed{\text{at most } 15}}$ mins. and has a uniform Prob. dist.



P(wait time is below 5 minutes) = P(x<5)

$= (5-0)\cdot\frac{1}{15} = \frac{5}{15} = \boxed{\frac{1}{3}}$

P(wait time $\underbrace{\text{is more than}}$ 12 minutes)

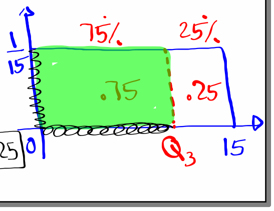
$= P(x>12) = (15-12)\cdot\frac{1}{15} = \frac{3}{15} = \boxed{\frac{1}{5}}$

Find Q_3 for the wait time.

75% below & 25% above

$(Q_3 - 0)\cdot\frac{1}{15} = .75$

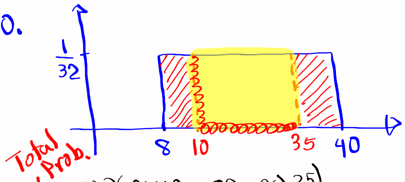
$Q_3 - 0 = 15(.75)$ $\boxed{Q_3 = 11.25}$



Jun 29-7:56 AM

Consider a uniform Prob. dist. for all values from 8 to 40.

1) Graph & label.



2) $P(x=10) = \boxed{0}$

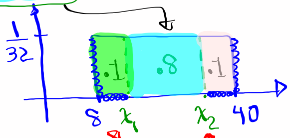
line
Zero Area

3) $P(x<10 \text{ OR } x>35)$

$= 1 - P(10<x<35)$

$= 1 - (35-10)\cdot\frac{1}{32} = 1 - \frac{25}{32} = \boxed{\frac{7}{32}}$

4) Find two values x_1 & x_2 that Separate the $\boxed{\text{middle } 80\%}$ from the rest.



Total Area = 1

1 - .8 = .2

.2÷2 = .1

$(x_1 - 8)\cdot\frac{1}{32} = .1$

$x_1 - 8 = 32(.1)$

$x_1 - 8 = 3.2$

$\boxed{x_1 = 11.2}$

$(40 - x_2)\cdot\frac{1}{32} = .1$

$40 - x_2 = 32(.1)$

$40 - x_2 = 3.2$

$40 - 3.2 = x_2$

$\boxed{x_2 = 36.8}$

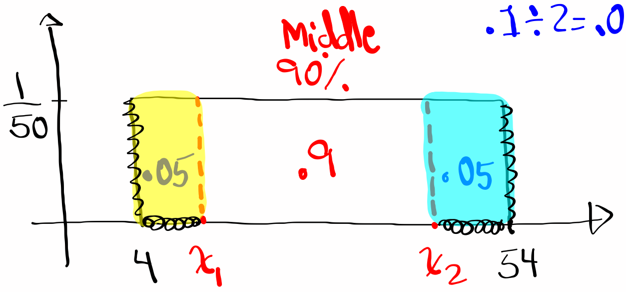
Jun 29-8:05 AM

Consider a Uniform Prob. dist. for all Values from 4 to 54.

$1-.9=.1$

$.1\div 2=.05$

1) Draw & label.



2) Find x_1 & x_2 that Separate the middle 90% from the rest.

$(x_1-4)\cdot\frac{1}{50}=.05$ $\quad x_1-4=50(.05)$ $\quad x_1-4=2.5$ $\quad \boxed{x_1=6.5}$

$(54-x_2)\cdot\frac{1}{50}=.05$ $\quad 54-x_2=50(.05)$ $\quad 54-x_2=2.5$ $\quad \boxed{x_2=51.5}$

Jun 29-8:16 AM

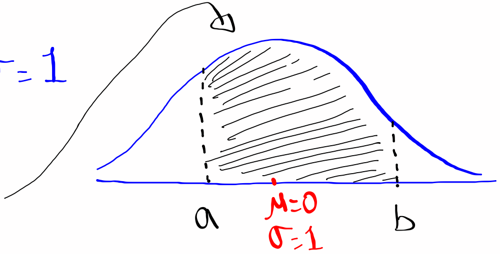
Standard Normal Prob. dist:

1) we use Z, $P(Z=c)=0$

2) Graph is symmetric, bell-shape with total area=1.

3) Mean = Mode = Median

4) $\mu=0$, $\sigma=1$



5) $P(a<Z<b)$

we use $\boxed{\text{2nd}}$ $\boxed{\text{VARS}}$ $\boxed{\text{normalcdf}}$ Command

$P(a<Z<b)=\text{normalcdf}(L, U, \mu, \sigma)$

Drawing, labeling, Shading, TI command required

Jun 29-8:37 AM

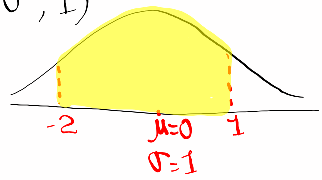
Find $P(-2 < Z < 1)$

$= \text{normalcdf}(\overset{(-)}{-2}, 1, \overset{\mu}{0}, \overset{\sigma}{1})$

(-)

Lower Upper

$= \boxed{.819}$

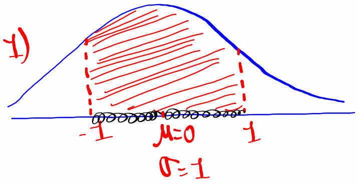


$P(-1 < Z < 1)$

$= \text{normalcdf}(-1, 1, 0, 1)$

$= \boxed{.683}$

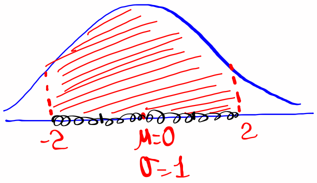
$\approx 68\%$



Find $P(-2 < Z < 2)$

$= \text{normalcdf}(-2, 2, 0, 1)$

$= \boxed{.954}$ $\approx 95\%$



Jun 29-8:43 AM

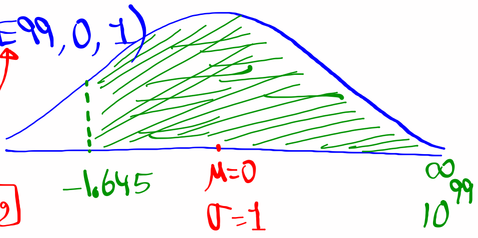
Find $P(Z > -1.645)$

$= \text{normalcdf}(-1.645, E99, 0, 1)$

(-)

$\boxed{2nd}\boxed{,}$

$= \boxed{.950}$



10^{99}

E99

$\boxed{2nd}\boxed{,}$

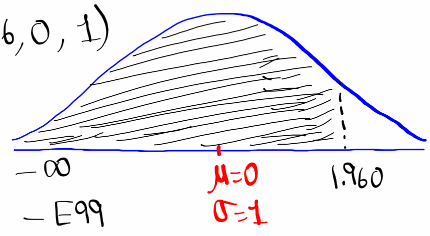
Find $P(Z < 1.960)$

$= \text{normalcdf}(-E99, 1.96, 0, 1)$

(-)

$\boxed{2nd}\boxed{,}$ $\boxed{,}$

$= \boxed{.975}$



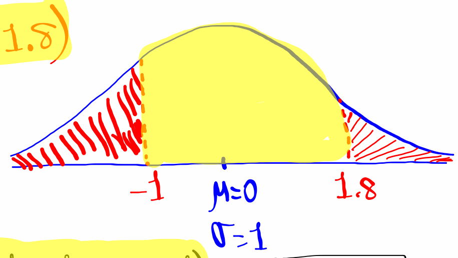
Jun 29-8:52 AM

Find $P(Z<-1 \boxed{\text{OR}} Z>1.8)$

$=1-P(-1<Z<1.8)$

↑

Total Area = Total Prob.

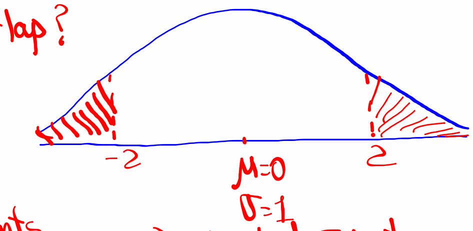


$=1-\text{normalcdf}(-1, 1.8, 0, 1) = \boxed{.195}$

Find $P(Z<-2 \boxed{\text{and}} Z>2) = \boxed{0}$

Is there an overlap?

NO



Mutually Exclusive Events

Disjointed Events

Jun 29-9:00 AM

Now doing Reverse

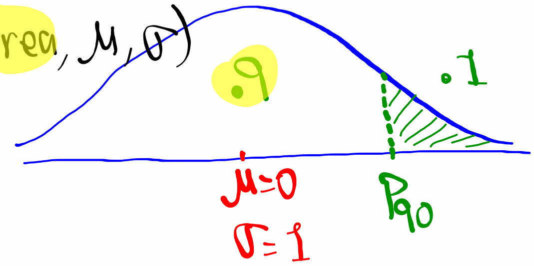
Find $Z = P_{90}$, Round to 3-decimal Places.

90% below & 10% above

$Z = P_{90} = \text{invNorm}(\text{Left Area}, \mu, \sigma)$

↑

$\boxed{\text{2nd}}\ \boxed{\text{VARS}}$



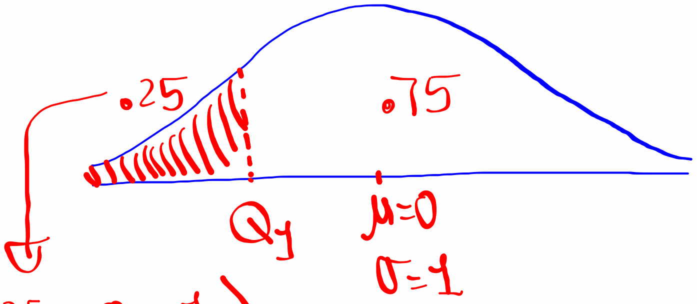
$Z = \text{invNorm}(.9, 0, 1)$

$= \boxed{1.282}$

Jun 29-9:08 AM

find $Z = Q_1$, Round to 3-decimals

25% below & 75% above



$Q_1 = \text{invNorm}(.25, 0, 1) = \boxed{-.674}$

Jun 29-9:12 AM

Find K such that $P(Z > K) = .01$

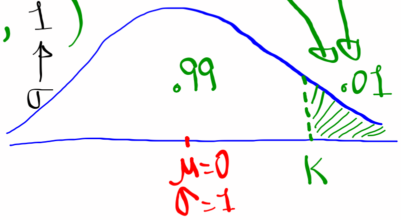
Right Area

$1 - .01 = .99$

$K = \text{invNorm}(.99, 0, 1)$

left Area, μ, σ

$= \boxed{2.326}$



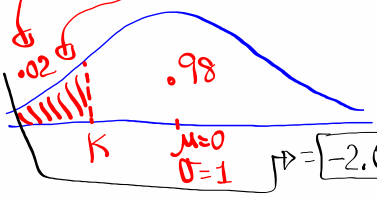
Find K such that $P(Z < K) = .02$

left Area

$1 - .02 = .98$

$K = \text{invNorm}(.02, 0, 1)$

left Area, μ, σ



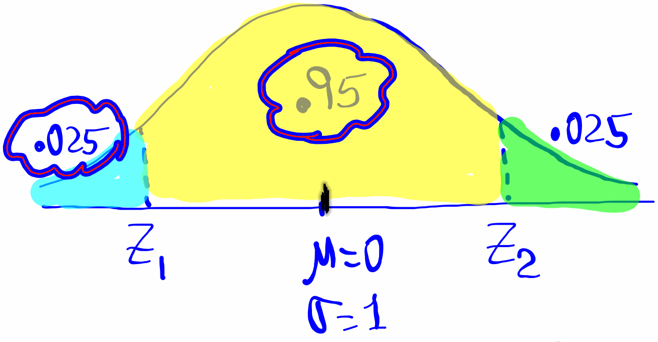
$= \boxed{-2.054}$

Jun 29-9:15 AM

Find two z-values, rounded to 3-decimal places, that separate the middle 95% from the rest.

$1 - .95 = .05$

$.05 \div 2 = .025$



$z_1 = \text{invNorm}(.025, 0, 1)$

$= \boxed{-1.960}$

left Area

$z_2 = \text{invNorm}(.975, 0, 1) = \boxed{1.960}$

Also by Symmetry

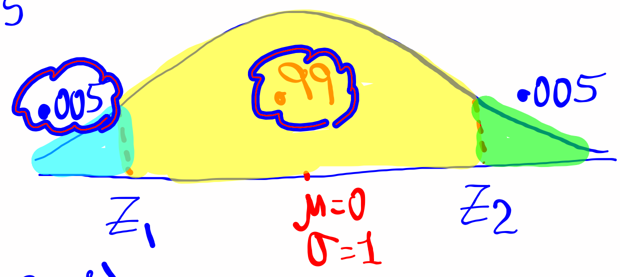
Jun 29-9:21 AM

Find two z-values, round to 3-decimal places, that separate the middle 99% from the rest.

$1 - .99 = .01$

Total Area

$.01 \div 2 = .005$



$z_1 = \text{invNorm}(.005, 0, 1)$

$= \boxed{-2.576}$

SG 18

$z_2 = \text{invNorm}(.995, 0, 1)$

$= \boxed{2.576}$

By Symmetry

Jun 29-9:28 AM

Normal Prob. Dist.: (SG19)

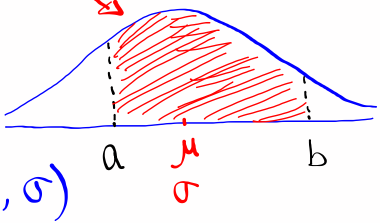
1) Use x, $P(x=c)=0$

2) Graph is bell-shape, symmetric with total area $=1$

3) Mean = Mode = Median

4) μ & σ is given in the process.

$P(a<x<b)$



$= \text{normalcdf}(L, U, \mu, \sigma)$

Normal → $N(\mu, \sigma)$

Mean

Stand. Dev.

Jun 29-9:58 AM

Given $N(75, 8)$

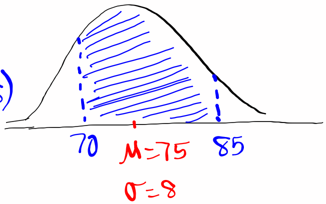
Normal Prob. Dist.

$\mu=75$ $\sigma=8$

Find $P(70<x<85)$

$= \text{normalcdf}(70,85,75,8)$

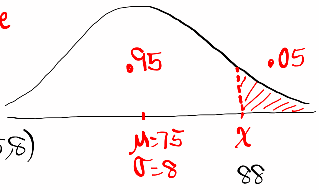
$= \boxed{.628}$



Find $x=P_{95}$, Round to a whole #

95% below, 5% above

Left Area .95 Right Area .05



$x=P_{95}=\text{invNorm}(.95,75,8)$

$=88.158$

$\approx \boxed{88}$

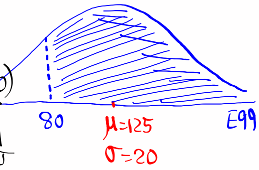
Jun 29-10:02 AM

Consider a normal Prob. dist. with $\mu = 125$ & $\sigma = 20$. $N(125, 20)$

$P(x > 80)$ $\boxed{2nd}\boxed{,}$

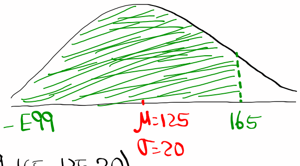
$= \text{normalcdf}(80, E99, 125, 20)$

L U μ σ



$= \boxed{.988}$

$P(x < 165)$



$= \text{normalcdf}(-E99, 165, 125, 20)$

(-) $\boxed{2nd}\boxed{,}$

$= \boxed{.977}$

Find $x = Q_1$, Round to whole #

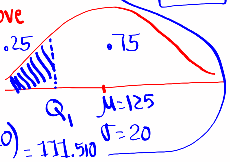
25% below & 75% above

Left Area .25

Right Area .75

$x = Q_1 = \text{invNorm}(.25, 125, 20) = 111.510$

$\boxed{112}$



Jun 29-10:08 AM

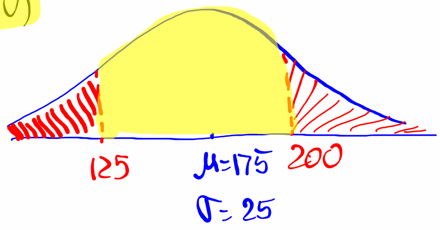
Consider a normal Prob. dist with the mean of 175 and Standard deviation of 25. $N(175, 25)$

$P(x < 125 \ \boxed{OR} \ x > 200)$

$= 1 - P(125 < x < 200)$

$= 1 - \text{normalcdf}(125, 200, 175, 25)$

$= \boxed{.181}$



$P(x < 125 \ \boxed{AND} \ x > 200) = 0$

No overlap

M.E.E. , Disjointed Events

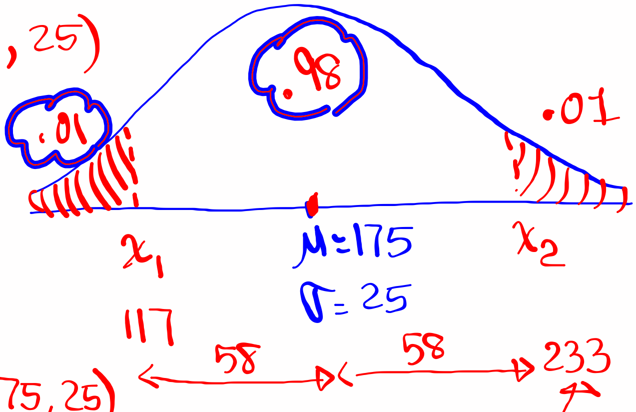
Jun 29-10:19 AM

Find two x-Values, Round to whole #, that Separate the middle 98% from the rest.

x_1 = invNorm(.01, 175, 25)

= 116.847

≈ $\boxed{117}$



x_2 = invNorm(.99, 175, 25)

= 233.159 ≈ $\boxed{233}$

Jun 29-10:25 AM

Exam Scores are normally dist. with $\mu=75$ and $\sigma=10$.

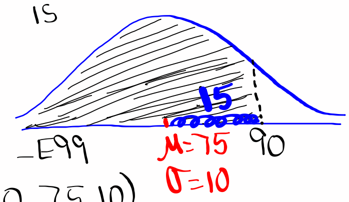
If we randomly Select one exam, find the prob. that Score x is

a) below 90.

$P(x<90)$

= normalcdf(−E99, 90, 75, 10)

= $\boxed{.933}$

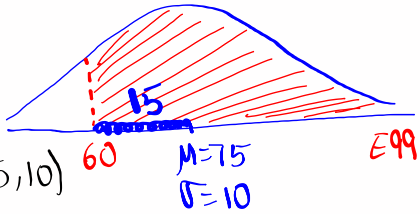


b) above 60.

$P(x>60)$

= normalcdf(60, E99, 75, 10)

= $\boxed{.933}$



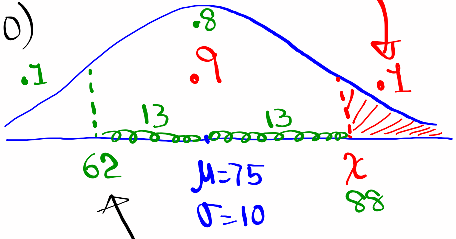
Jun 29-10:32 AM

3) Find $\overbrace{\text{exam Score}}^{x}$, round to whole #, that Separates the top 10% from the rest.

$x = \text{invNorm}(.9, 75, 10)$

$= 87.816$

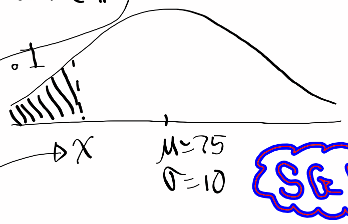
$\approx \boxed{88}$



Find the Score that Separates the bottom 10% from the rest, Round to whole #.

$x = \text{invNorm}(.1, 75, 10)$

$= 62.184 \approx \boxed{62}$



SG-19

Jun 29-10:41 AM

Review for SG 17

Consider a geometric prob. dist. with p=.9

$q = 1 - p = 1 - .9 = \boxed{.1}$

$\mu = \frac{1}{p} = \frac{1}{.9} = \boxed{1.111}$

$\sigma = \sqrt{\sigma^2}$

$= \sqrt{.123} = \boxed{.351}$

$\sigma^2 = \frac{q}{p^2} = \frac{.1}{.9^2} = \boxed{.123}$

$\mu \approx 1.1, \ \sigma \approx .4$

Usual Range

$\mu \pm 2\sigma =$

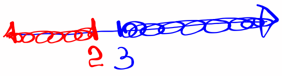
$1.1 \pm 2(.4) \Rightarrow \boxed{.3 \text{ to } 1.9}$

P(First Success happens on 3rd trial) =

$P(x=3) = \text{geomet pdf}(.9, 3) = \boxed{.009}$

P(first Success happens after 2nd trial):

$P(x>2) = P(x \geq 3) = 1 - P(x \leq 2)$



$= 1 - \text{geometcdf}(.9, 2)$

$= \boxed{.01}$

Jun 29-11:08 AM

Lisa serves 16 people in any hour at work in average

Fixed Interval

$\mu=16$

Poisson Dist.

$\sigma^2 = \mu = 16$ $\sigma = \sqrt{\sigma^2} = 4$

68% Range $\Rightarrow \mu \pm \sigma = 16 \pm 4 \Rightarrow \boxed{12 \text{ To } 20}$

Find the prob. that she serves between 12 and 20, inclusive, customers per hour.

$P(12 \leq x \leq 20) = \text{Poissoncdf}(16,20) - \text{Poissoncdf}(16,11)$

$= \boxed{.741}$

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1 in 40 packages arrive late. $P = \frac{1}{40}$

P(First late arrival happens on 4th delivery)

$P(x=4) = \text{geomet pdf}(1/40, 4) = .023$

P(First late delivery happens before the 10th delivery)

$P(x<10) = P(x \leq 9) = \text{geometcdf}(1/40, 9)$

$= \boxed{.204}$

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